

8.5 Greyhounds Chasing Greyhounds

WORKSHEET

The question

Four greyhounds A , B , C and D are held at positions such that $ABCD$ is a large square. At a given instant, the dogs are released and A runs directly towards B at constant speed v , B runs directly towards C at constant speed v , and so on. Show that A 's path is given in polar coordinates (referred to an origin at the centre of the field and a suitable initial line) by $r = \lambda e^{-\theta}$, where λ is a constant.

Generalise this result to the case of n dogs held at the vertices of a regular n -gon ($n \geq 3$).

Hints

Try the question on your own first. If you get stuck, read the hints *one at a time*, and go back to the question after each one. Hints marked *Pause and try* are the key steps.



Ask yourself By symmetry, what shape do the four dogs form at every moment? Where is its centre?

STAGE 1 The direction of motion



Ask yourself A runs towards B . In the triangle OAB , what angle does AB make with AO (the direction towards the centre)?

STAGE 2 The differential equation



Ask yourself Split A 's velocity into radial and transverse parts. In polar coordinates the radial velocity is $\dot{r}$ and the transverse velocity is $r\dot{\theta}$.

STAGE 3 n dogs



Pause and try For a regular n -gon, what is the angle at O in triangle OAB ? What angle does AB make with AO ? Repeat Stage 2.

Try this next

- 1 The slick argument.** Explain carefully why the distance AB decreases at rate exactly v in the four-dog problem, and deduce that the dogs meet after time s/v .
- 2 Three dogs.** Three dogs start at the corners of an equilateral triangle of side s . Find the equation of each path, the time until they meet, and the distance each runs.
- 3 How many turns?** For four dogs starting 100 m from the centre, how close to the centre are they after one full turn? After how many full turns are they within 1 mm?
- 4 Two dogs.** What happens with $n = 2$ (two dogs facing each other)? Does the formula still make sense?

Further reading

- **Paul Nahin, *Chases and Escapes: The Mathematics of Pursuit and Evasion*** (Princeton University Press). Everything about pursuit curves, from Bouguer to missiles.
- **Eli Maor, *e: The Story of a Number*** (Princeton University Press). Includes the logarithm-

mic spiral and Bernoulli's tombstone.

- **D'Arcy Thompson, *On Growth and Form*** (Cambridge University Press). The classic book on mathematics in nature, including spiral growth.

The full guided solution, and what the question leads to at university, is in Section 8.5:
https://keithmcnulty.github.io/step_book_mcnulty/ch08/pursuit.html