

13.2 Counting Voles

WORKSHEET

The question

A group of biologists attempts to estimate the magnitude, N , of an island population of voles (*Microtus agrestis*). Accordingly, the biologists capture a random sample of 200 voles, mark them and release them. A second random sample of 200 voles is then taken of which 11 are found to be marked. Show that the probability, p_N , of this occurrence is given by

$$p_N = k \frac{((N - 200)!)^2}{N! (N - 389)!},$$

where k is independent of N .

The biologists then estimate N by calculating the value of N for which p_N is a maximum. Find this estimate.

All unmarked voles in the second sample are marked and then the entire sample is released. Subsequently a third random sample of 200 voles is taken. Write down the probability that this sample contains exactly j marked voles, leaving your answer in terms of binomial coefficients.

Deduce that

$$\sum_{j=0}^{200} \binom{389}{j} \binom{3247}{200-j} = \binom{3636}{200}.$$

Hints

Try the question on your own first. If you get stuck, read the hints *one at a time*, and go back to the question after each one. Hints marked *Pause and try* are the key steps.



Ask yourself The population has N voles, 200 of them marked. A random sample of 200 is taken. What is the probability that exactly 11 are marked?

STAGE 2 Maximising over a whole number



Ask yourself N is a whole number, so calculus is awkward. Instead compare p_{N+1} with p_N . When is their ratio bigger than 1?

Try this next

- 1 **General formula.** If M animals are marked and a second sample of C contains R marked, show that the maximum likelihood estimate of N is the integer part of $\frac{MC}{R}$.
- 2 **Vandermonde.** Prove $\sum_j \binom{a}{j} \binom{b}{c-j} = \binom{a+b}{c}$ by counting committees of c people chosen from a women and b men.
- 3 **Coin tosses.** In n tosses of a coin with unknown probability p of heads, h heads are observed. Show that the maximum likelihood estimate of p is $\frac{h}{n}$.
- 4 **No recaptures.** What happens to the maximum likelihood estimate if the second sample contains no marked voles? What does that tell you about the method?

Further reading

- **David Spiegelhalter, *The Art of Statistics*** (Pelican), on estimation and uncertainty.
- **George Seber, *The Estimation of Animal Abundance*** (Blackburn). The standard reference on capture–recapture.
- **John Rice, *Mathematical Statistics and Data Analysis*** (Duxbury). A first course in mathematical statistics, including maximum likelihood.

The full guided solution, and what the question leads to at university, is in Section 13.2:
https://keithmcnulty.github.io/step_book_mcnulty/ch13/voles.html