

12.13 The Truel

WORKSHEET

The question

Bread roll throwing duels at the Drones' Club are governed by a strict etiquette. The two duellists throw alternately until one is hit, when the other is declared the winner. If Percy has probability $p > 0$ of hitting his target and Rodney has probability $r > 0$ of hitting his, show that, if Percy throws first, the probability that he beats Rodney is

$$\frac{p}{p + r - pr}.$$

Algernon, Bertie and Cuthbert decide to have a three sided duel in which they throw in order A, B, C, A, B, C, ... except that anyone who is hit must leave the game. Cuthbert always hits his target, Bertie hits his target with probability $\frac{3}{5}$ and Algernon hits his target with probability $\frac{2}{5}$. Bertie and Cuthbert will always aim at each other if they are both still in the duel. Otherwise they aim at Algernon. With his first shot Algernon may aim at either Bertie or Cuthbert or deliberately miss both. Faced with only one opponent Algernon will aim at him. What are Algernon's chances of winning if he:

- (i) hits Cuthbert with his first shot?
- (ii) hits Bertie with his first shot?
- (iii) misses with his first shot?

Advise Algernon as to his best plan and show that, if he uses this plan, his probability of winning is $226/475$.

Hints

Try the question on your own first. If you get stuck, read the hints *one at a time*, and go back to the question after each one. Hints marked *Pause and try* are the key steps.

- ? Ask yourself** In a two-person duel, Percy throws first. Either he hits at once, or both miss and the situation repeats exactly. Write an equation for Percy's probability of winning.

STAGE 2 Algernon's three options

- ? Ask yourself** In each case, who is left, and who throws next?

STAGE 3 Advice

- ? Ask yourself** If Algernon aims at Cuthbert, he might hit (probability $\frac{2}{5}$) or miss. What is his overall chance? Compare with deliberately missing.

Try this next

- 1 Bertie and Cuthbert.** Verify that, with Algernon missing deliberately, Bertie wins with probability $\frac{135}{475}$ and Cuthbert with $\frac{114}{475}$.
- 2 A better shot.** If Algernon hit with probability a instead of $\frac{2}{5}$, for which values of a is deliberately missing still his best first move?
- 3 Should Bertie aim at Cuthbert?** Given that Algernon misses deliberately, is it in Bertie's interest to aim at Cuthbert rather than at Algernon? (Assume Cuthbert's behaviour is unchanged.)
- 4 A duel with a fixed number of throws.** In a duel where each player has only two throws, find Percy's probability of winning, and compare with the unlimited duel.

Further reading

- **D. Marc Kilgour & Steven Brams**, "The truel", *Mathematics Magazine* 70 (1997). An accessible survey of three-way duels.
- **Avinash Dixit & Barry Nalebuff**, *Thinking Strategically* (Norton). Game theory for a general reader, including the truel.
- **Ken Binmore**, *Game Theory: A Very Short Introduction* (Oxford University Press).

The full guided solution, and what the question leads to at university, is in Section 12.13:

https://keithmcnulty.github.io/step_book_mcnulty/ch12/truel.html