

12.1 Precise Conditioning

WORKSHEET

The question

A bag contains eleven small discs, which are identical except that six of the discs are blank and five of the discs are numbered, using the numbers 1, 2, 3, 4 and 5. The bag is shaken, and four discs are taken one at a time without replacement.

Calculate the probability that:

- (i) all four discs taken are numbered;
- (ii) all four discs taken are numbered, given that the disc numbered “3” is taken first;
- (iii) exactly two numbered discs are taken, given that the disc numbered “3” is taken first;
- (iv) exactly two numbered discs are taken, given that the disc numbered “3” is taken;
- (v) exactly two numbered discs are taken, given that a numbered disc is taken first;
- (vi) exactly two numbered discs are taken, given that a numbered disc is taken.

Hints

Try the question on your own first. If you get stuck, read the hints *one at a time*, and go back to the question after each one. Hints marked *Pause and try* are the key steps.



Ask yourself Four discs are taken from eleven. Does the order of drawing matter for the event “all four are numbered”? How many equally likely sets of four discs are there?

STAGE 2 The 3 comes first



Ask yourself Once the disc numbered 3 has been drawn first, what are the remaining three draws?

STAGE 3 The 3 is taken (at some point)



Ask yourself Does it matter *when* the disc numbered 3 was drawn? Given that 3 is among the four, what can you say about the other three?

STAGE 5 A numbered disc is taken



Ask yourself Now the condition is “at least one of the four is numbered”. Is that the same as any of the previous conditions? How many of the 330 sets satisfy it?



Pause and try Why is (vi) different from (v)? In (v) we know *which* disc is numbered (the first), and the other three are unconstrained. In (vi) we only know that *some* disc is numbered. Sets with three or four numbered discs satisfy “some disc is numbered” in more ways, so they are not under-represented the way they are in (v).

Try this next

- Generalise.** With b blank and n numbered discs, and k drawn, show that the answers to (iv) and (v) are always equal, and find when (vi) equals them.
- Boys born on Tuesday.** A family has two children, and at least one is a boy born on a Tuesday. Show that the probability that both are boys is $\frac{13}{27}$. (List the 14×14 equally likely possibilities, or count cleverly.)
- Monty Hall.** Verify the $\frac{2}{3}$ by listing outcomes. What if the host does not know where the car is, opens one of the other doors at random, and it happens to show a goat?

- 4 **Two aces.** A bridge hand of 13 cards contains at least one ace. What is the probability that it contains at least two? Compare with the probability given that it contains the ace of spades.

Further reading

- ▶ **Jason Rosenhouse, *The Monty Hall Problem*** (Oxford University Press). The puzzle, its history and its many variations.
- ▶ **Geoffrey Grimmett & Dominic Welsh, *Probability: An Introduction*** (Oxford University Press), the chapters on conditional probability.
- ▶ **Joseph Blitzstein & Jessica Hwang, *Introduction to Probability*** (CRC Press; free online). Excellent on conditioning, with many paradoxes.

The full guided solution, and what the question leads to at university, is in Section 12.1:
https://keithmcnulty.github.io/step_book_mcnulty/ch12/conditioning.html