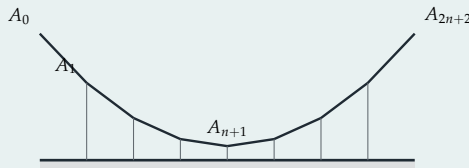


11.14 Parabola, Not Catenary

WORKSHEET

The question



The above diagram represents a suspension bridge. A heavy uniform horizontal roadway is attached by vertical struts to a light flexible chain at points $A_1 = (x_1, y_1)$, $A_2 = (x_2, y_2), \dots, A_{2n+1} = (x_{2n+1}, y_{2n+1})$, where the coordinates are referred to horizontal and vertically upward axes Ox, Oy . The chain is fixed to external supports at points

$$A_0 = (x_0, y_0) \quad \text{and} \quad A_{2n+2} = (x_{2n+2}, y_{2n+2})$$

at the same height. The weight of the chain and struts may be neglected. Each strut carries the same weight w . The horizontal spacing h between A_i and A_{i+1} (for $0 \leq i \leq 2n+1$) is constant. Write down equations satisfied by the tensions T_i in the portion $A_{i-1}A_i$ of the chain for $1 \leq i \leq n+1$. Hence or otherwise show that

$$\frac{h}{y_n - y_{n+1}} = \frac{3h}{y_{n-1} - y_n} = \dots = \frac{(2n+1)h}{y_0 - y_1}.$$

Verify that the points $A_0, A_1, \dots, A_{2n+1}, A_{2n+2}$ lie on a parabola.

Hints

Try the question on your own first. If you get stuck, read the hints *one at a time*, and go back to the question after each one. Hints marked *Pause and try* are the key steps.

- ?** **Ask yourself** At each joint A_i , three forces act: the tensions in the two neighbouring segments, and the weight w hanging from the strut. Which component of tension can the weight not affect?

STAGE 1 The equations at each joint

- ?** **Ask yourself** What happens at the middle joint A_{n+1} ? Use the symmetry of the bridge.

STAGE 3 The parabola

- ?** **Ask yourself** Let j count joints outwards from the middle, so that A_{n+1-j} is at horizontal distance jh from A_{n+1} . How far is it above A_{n+1} ?

Try this next

- 1 An even number of struts.** Repeat the question with $2n$ struts, so that the lowest point of the chain is a horizontal segment in the middle. Do the joints still lie on a parabola?
- 2 The tension.** Find the greatest tension in the chain, in terms of H , w and n . Where is it?
- 3 The catenary.** Verify that $y = c \cosh \frac{x}{c}$ satisfies $y'' = \frac{1}{c} \sqrt{1 + y'^2}$. Show that the length of the catenary from $x = 0$ to $x = X$ is $c \sinh \frac{X}{c}$.
- 4 Galileo's error.** A chain of length $2L$ hangs between two supports at distance $2d$ apart. For L only slightly larger than d , show that the sag of the catenary is approximately the same as that of a parabola. Why did Galileo's experiments not reveal the difference?

Further reading

- **E. H. Lockwood, *A Book of Curves*** (Cambridge University Press). The catenary, the parabola and their relatives.
- **Jacques Heyman, *The Stone Skeleton*** (Cambridge University Press). How masonry arches and domes stand up, from Hooke's chain onwards.
- **David Billington, *The Tower and the Bridge*** (Princeton University Press). The art and

engineering of bridges.

The full guided solution, and what the question leads to at university, is in Section 11.14:
https://keithmcnulty.github.io/step_book_mcnulty/ch11/bridge-parabola.html