

11.10 A Ball That Comes Back Towards You

WORKSHEET

The question

A particle P is projected from a point O on horizontal ground with speed u and angle of projection α , where $0 < \alpha < \frac{1}{2}\pi$.

- (i) Show that if $\sin \alpha < \frac{2\sqrt{2}}{3}$, then the distance OP is increasing throughout the flight.

Show also that if $\sin \alpha > \frac{2\sqrt{2}}{3}$, then OP will be decreasing at some time before the particle lands.

- (ii) At the same time as P is projected, a particle Q is projected horizontally from O with speed v along the ground in the opposite direction from the trajectory of P . The ground is smooth. Show that if

$$2\sqrt{2}v > (\sin \alpha - 2\sqrt{2}\cos \alpha)u,$$

then QP is increasing throughout the flight of P .

Hints

Try the question on your own first. If you get stuck, read the hints *one at a time*, and go back to the question after each one. Hints marked *Pause and try* are the key steps.



Ask yourself Why is it easier to work with OP^2 than with OP ? Is “ OP is increasing” the same as “ OP^2 is increasing”?

STAGE 1 The squared distance



Ask yourself Differentiate with respect to t . After taking out a factor of t , what is left? When is it positive for all t ?

STAGE 2 Steeper throws



Ask yourself If $\sin \alpha > \frac{2\sqrt{2}}{3}$, the quadratic has two real roots. Are they positive? Is the smaller one before the particle lands?

STAGE 3 A moving observer



Ask yourself Q moves away from O along the ground in the opposite direction, at speed v . Measured from Q , the horizontal separation is $x + vt$. What does that do to the calculation?

Try this next

- 1 How close?** For $\alpha = 80^\circ$, find the times at which OP has its local maximum and minimum, and the distances there, in units of u^2/g .
- 2 On a slope.** Repeat part (i) for a ball thrown up a plane inclined at angle β to the horizontal, landing on the plane. Does the critical angle (measured from the plane) change?
- 3 With air resistance.** Suppose the ball also experiences a resistance proportional to its velocity. Without calculating, would you expect the critical angle to be larger or smaller than 70.5° ? Why?
- 4 Throwing from a height.** If the ball is thrown from the top of a cliff and lands far below, show that OP eventually increases for any angle, and find whether there are angles for which it decreases at some stage.

Further reading

- ▶ **James Walker**, *The Flying Circus of Physics* (Wiley). Hundreds of surprising everyday physics puzzles.
- ▶ **David Morin**, *Introduction to Classical Mechanics* (Cambridge University Press), Chapter 3, on projectiles, with several counter-intuitive problems.
- ▶ **J. M. Steele**, *The Cauchy–Schwarz Master Class* (Cambridge University Press). The art of proving that one thing is always bigger than another.

The full guided solution, and what the question leads to at university, is in Section 11.10:
https://keithmcnulty.github.io/step_book_mcnulty/ch11/coming-back.html