

10.2 The Stacked-Ball Effect

WORKSHEET

The question

- (i) In an experiment, a particle A of mass m is at rest on a smooth horizontal table. A particle B of mass bm , where $b > 1$, is projected along the table directly towards A with speed u . The collision is perfectly elastic.

Find an expression for the speed of A after the collision in terms of b and u , and show that, irrespective of the relative masses of the particles, A cannot be made to move at twice the initial speed of B .

- (ii) In a second experiment, a particle B_1 is projected along the table directly towards A with speed u . This time, particles $B_2, B_3, \dots, B_n$ are at rest in order on the line between B_1 and A . The mass of B_i ($i = 1, 2, \dots, n$) is $\lambda^{n+1-i}m$, where $\lambda > 1$. All collisions are perfectly elastic. Show that, by choosing n sufficiently large, there is no upper limit on the speed at which A can be made to move.

In the case $\lambda = 4$, determine the least value of n for which A moves at more than $20u$. You may use the approximation $\log_{10} 2 \approx 0.30103$.

Hints

Try the question on your own first. If you get stuck, read the hints *one at a time*, and go back to the question after each one. Hints marked *Pause and try* are the key steps.



Ask yourself From Section 10.1: if a mass M moving at speed u hits a stationary mass m elastically, what is the speed of m afterwards?

STAGE 1 The speed limit



Ask yourself How does $\frac{2b}{b+1}$ compare with 2? What happens as $b \rightarrow \infty$?

STAGE 2 Passing it along a chain



Ask yourself In part (ii), each particle is λ times as heavy as the next one. What does part (i) say about each collision in the chain?



Ask yourself Is $\frac{2\lambda}{\lambda+1}$ bigger or smaller than 1?

Try this next

- 1 The real thing.** (From STEP I, 2014.) A ball of mass M and radius R is released from rest with its centre at height $H + R$, with a ball of mass m and radius r on top. The coefficient of restitution for every impact is e . Show that, bouncing alone, the large ball would rise until its centre is at height $R + He^2$. Given $R = 0.2$, $r = 0.05$, $H = 1.8$, $e = \frac{2}{3}$ and that the small ball's centre reaches height 4.5, show that $M/m = 9$.
- 2 An inelastic chain.** (From STEP I, 2017.) Particles $P_1, P_2, \dots$ of masses $\lambda m, \lambda^2 m, \dots$ lie at $x = 1, 2, \dots$, and a particle of mass m hits P_1 at speed u ; the coefficient of restitution is e . Show that P_1 moves off at $\frac{1+e}{1+\lambda}u$. Find the speed of P_n after its first collision, and show that if $e > \lambda$ each particle is involved in exactly two collisions. What happens if $\lambda e = 1$?
- 3 Three balls.** Three perfectly elastic balls are stacked, each much lighter than the one below. Show that the top ball leaves at $7U$ and rises to $49H$. With k balls, show that the top one leaves at $(2^k - 1)U$.
- 4 Where does the energy come from?** In part (ii), show that the total kinetic energy is conserved even though A 's speed is unbounded. What happens to the heavy particles?

Further reading

- ▶ **David Morin**, *Introduction to Classical Mechanics* (Cambridge University Press), Chapter 5, which includes the stacked-ball problem.
- ▶ **W. R. Mellen**, “**Superball rebound projectiles**”, *American Journal of Physics* 36 (1968). Short and fun.
- ▶ **Gary Flandro**, “**Fast reconnaissance missions to the outer solar system utilizing energy derived from the gravitational field of Jupiter**”, *Astronautica Acta* 12 (1966). The paper that proposed the Voyager “grand tour”.

The full guided solution, and what the question leads to at university, is in Section 10.2:
https://keithmcnulty.github.io/step_book_mcnulty/ch10/stacked-balls.html