Mathematical Symbols

Mathematical writing packs a lot of meaning into a few marks, and university lecturers use many symbols that school textbooks rarely do. The tables below list the symbols used in this book, grouped by topic, with what each one means and how it is read aloud. Most of them will become second nature within a few weeks of starting a degree; until then, this is somewhere to look them up.

Two conventions are worth knowing from the start. First, this book follows STEP in printing some letters upright rather than in italics when they have a fixed meaning: e for the base of natural logarithms, i for $\sqrt{-1}$, d in $\mathrm{d}y/\mathrm{d}x$, and the names of functions, as in $\mathrm{f}(x)$ and $\mathrm{P}(A)$. Italic letters are variables. Secondly, the same symbol sometimes means different things in different contexts; $\equiv$ and $\sim$ are the main culprits, and the tables point out where.

Logic and proof

Symbol	Meaning	Example or reading
$P \Rightarrow Q$	P implies Q: if P is true, so is Q	$x = 2 \Rightarrow x^2 = 4$; "$P$ implies Q"
$P \Leftarrow Q$	P is implied by Q	"P if Q"
$P \iff Q$	P if and only if Q: each implies the other	$x^2 = 4 \iff x = \pm 2$; also written "iff"
$\forall$	for all, for every	$\forall x \in \mathbb{R},\ x^2 \geq 0$
$\exists$	there exists	$\exists\, n \in \mathbb{Z}$ with $n^2 = 9$
$\neg P$	not P, the negation of P	the negation of "$\forall x,\ P(x)$" is "$\exists\, x$ with $\neg P(x)$"
$:=$	is defined to equal	$\mathrm{f}(x) := x^2 + 1$
$\blacksquare$	end of proof; also written QED	from the Latin *quod erat demonstrandum*, "which was to be shown"

Sets and number systems

Symbol	Meaning	Example or reading
$\{\dots\}$	the set whose elements are listed	$\{1, 2, 3\}$
$\{x : \dots\}$	the set of all x such that $\dots$; also written $\{x \mid \dots\}$	$\{x : x^2 < 2\}$; "the set of x such that $x^2 < 2$"
$x \in A$	x is an element of A	$\sqrt{2} \in \mathbb{R}$; "$\sqrt{2}$ is in $\mathbb{R}$"
$x \notin A$	x is not an element of A	$\sqrt{2} \notin \mathbb{Q}$
$A \subseteq B$	A is a subset of B: every element of A is in B	$\mathbb{Z} \subseteq \mathbb{Q}$
$A \cup B$	union: elements in A or B (or both)	$\{1, 2\} \cup \{2, 3\} = \{1, 2, 3\}$
$A \cap B$	intersection: elements in both A and B	$\{1, 2\} \cap \{2, 3\} = \{2\}$
$A \setminus B$	elements of A that are not in B	$\{1, 2, 3\} \setminus \{2\} = \{1, 3\}$
A'	complement: everything not in A	also written A^{c} or $\bar{A}$
$\varnothing$	the empty set, with no elements	$\{x \in \mathbb{R} : x^2 < 0\} = \varnothing$
$\lvert A \rvert$	the number of elements of a finite set A	$\lvert\{a, b, c\}\rvert = 3$
$A \times B$	the set of all pairs (a, b) with $a \in A$, $b \in B$	$\mathbb{R} \times \mathbb{R}$ is the plane, also written $\mathbb{R}^2$
$\mathbb{N}$	the natural numbers $1, 2, 3, \dots$	some authors include 0
$\mathbb{Z}$	the integers $\dots, -2, -1, 0, 1, 2, \dots$	from the German *Zahlen*, "numbers"
$\mathbb{Q}$	the rational numbers (fractions)	from *quotient*
$\mathbb{R}$	the real numbers	
$\mathbb{C}$	the complex numbers	
$\mathbb{H}$	the quaternions	after Hamilton (Section 6.7)
$\mathbb{Z}[\mathrm{i}]$	numbers $a + b\mathrm{i}$ with $a, b \in \mathbb{Z}$	the Gaussian integers (Section 1.6)
$[a, b]$	closed interval: $a \le x \le b$	includes its endpoints
(a, b)	open interval: $a < x < b$	excludes its endpoints; also a pair or a point

Numbers, divisibility and approximation

Symbol	Meaning	Example or reading
$a \mid b$	a divides b: b is an integer multiple of a	$3 \mid 12$; "3 divides 12"
$a \nmid b$	a does not divide b	$5 \nmid 12$
$a \equiv b \pmod{n}$	a and b leave the same remainder on division by n	$17 \equiv 2 \pmod{5}$; "17 is congruent to 2 modulo 5"
$\equiv$	(without a modulus) is identically equal to: true for all values	$(x+1)^2 \equiv x^2 + 2x + 1$
$\lfloor x \rfloor$	floor: the greatest integer $\leq x$	$\lfloor 3.7 \rfloor = 3$, $\lfloor -3.7 \rfloor = -4$
$\lceil x \rceil$	ceiling: the least integer $\geq x$	$\lceil 3.2 \rceil = 4$
$n!$	n factorial, $1 \times 2 \times \cdots \times n$	$0! = 1$ by convention
$\binom{n}{r}$	the binomial coefficient $\frac{n!}{r!\,(n-r)!}$	"n choose r"; also written ${}^{n}\mathrm{C}_{r}$
$\approx$	is approximately equal to	$\pi \approx 3.1416$
$\sim$	is asymptotic to: the ratio tends to 1	$n! \sim \sqrt{2\pi n}\,(n/\mathrm{e})^n$ (but see also Probability)
$\propto$	is proportional to	$v \propto 1/d$
$\pm$, $\mp$	plus or minus; minus or plus	$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
$0.\overline{01}$	recurring digits	$0.\overline{01} = 0.010101\ldots$

Functions

Symbol	Meaning	Example or reading
$\mathrm{f} : A \to B$	f is a function from the set A to the set B	$\mathrm{f} : \mathbb{R} \to \mathbb{R}$
$x \mapsto \ldots$	x is sent to $\ldots$	$x \mapsto x^2 - 2$; "x maps to $x^2 - 2$"
$\mathrm{f} \circ \mathrm{g}$	composition: do g, then f	$(\mathrm{f} \circ \mathrm{g})(x) = \mathrm{f}(\mathrm{g}(x))$
f^{-1}	the inverse function of f	not $1/\mathrm{f}$
f^n	in this book, usually f applied n times	$\mathrm{f}^3(x) = \mathrm{f}(\mathrm{f}(\mathrm{f}(x)))$; but $\sin^2 x$ means $(\sin x)^2$
$\lvert x \rvert$	modulus, or absolute value	$\lvert -3 \rvert = 3$
$\exp x$	the exponential function e^x	
$\ln x$	natural logarithm, to base e	at university often written $\log x$
sec, cosec, cot	1/ cos, 1/ sin, 1/ tan	
arcsin, arccos, arctan	inverse trigonometric functions	also written $\sin^{-1}$, etc.
sinh, cosh, tanh	hyperbolic functions: $\sinh x = \frac{1}{2}(\mathrm{e}^x - \mathrm{e}^{-x})$, $\cosh x = \frac{1}{2}(\mathrm{e}^x + \mathrm{e}^{-x})$, tanh = sinh / cosh	read "shine" (or "sinch"), "cosh" and "than" (or "tanch")
artanh	inverse of tanh	$\operatorname{artanh} x = \frac{1}{2} \ln \frac{1+x}{1-x}$
max, min	the largest, smallest of	$\max(3, 7) = 7$
$\Gamma(x)$	the gamma function	$\Gamma(n+1) = n!$ (Section 4.4)

Sums, limits and calculus

Symbol	Meaning	Example or reading
$\sum$	sum	$\sum_{k=1}^{n} k = \frac{1}{2}n(n+1)$
$\prod$	product	$\prod_{k=1}^{n} k = n!$
∞	infinity: not a number, but shorthand for "without limit"	$\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6}$
$x \to a$	x tends to a	$\frac{1}{n} \to 0$ as $n \to \infty$
lim	limit	$\lim_{x\to 0} \frac{\sin x}{x} = 1$
$\mathrm{f}'(x)$, $\mathrm{f}''(x)$	first and second derivatives	"f dash", or "f prime"
$\dot{x}$, $\ddot{x}$	derivatives with respect to time	$\dot{x} = \frac{\mathrm{d}x}{\mathrm{d}t}$; "$x$ dot", "x double dot"
$\frac{\partial f}{\partial x}$	partial derivative: differentiate with respect to x, holding other variables fixed	"partial f by partial x"
$\int_a^b$	definite integral from a to b	
$\oint$	integral round a closed loop	$\oint p\,\mathrm{d}q$ (Section 10.4)
δx, Δx	a small change in x; a change in x	
ε, δ	conventional names for small positive numbers in analysis	"for every $\varepsilon > 0$ there is a $\delta > 0$ such that ..."

Complex numbers

Symbol	Meaning	Example or reading
i	a square root of -1	engineers write j
$\mathrm{Re}\,z$, $\mathrm{Im}\,z$	real and imaginary parts	if $z = 3 + 4\mathrm{i}$, $\mathrm{Re}\,z = 3$, $\mathrm{Im}\,z = 4$
z^*, $\bar{z}$	complex conjugate	$(3+4\mathrm{i})^* = 3 - 4\mathrm{i}$; both notations are used
$\lvert z\rvert$	modulus: distance from 0	$\lvert 3+4\mathrm{i}\rvert = 5$
$\arg z$	argument: angle from the positive real axis	$\arg \mathrm{i} = \frac{\pi}{2}$
$\mathrm{e}^{\mathrm{i}\theta}$	$\cos\theta + \mathrm{i}\sin\theta$	$\mathrm{e}^{\mathrm{i}\pi} = -1$
ω	conventionally, a complex cube root of unity	$\omega = \mathrm{e}^{2\pi\mathrm{i}/3}$, $1 + \omega + \omega^2 = 0$

Vectors and matrices

Symbol	Meaning	Example or reading
$\mathbf{a}$, $\overrightarrow{AB}$	a vector; the vector from A to B	handwritten as $\underline{a}$
$\hat{\mathbf{n}}$	a unit vector, of length 1	"n hat"
$\lvert\mathbf{a}\rvert$	the length (magnitude) of $\mathbf{a}$	
$\mathbf{a}\cdot\mathbf{b}$	scalar (dot) product	$\mathbf{a}\cdot\mathbf{b} = \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta$
$\mathbf{a}\times\mathbf{b}$	vector (cross) product	perpendicular to $\mathbf{a}$ and $\mathbf{b}$
$\mathbf{M}$	a matrix	
$\mathbf{I}$	the identity matrix	$\mathbf{Iv} = \mathbf{v}$ for every $\mathbf{v}$
$\mathbf{M}^{-1}$	the inverse matrix	$\mathbf{MM}^{-1} = \mathbf{I}$
$\det\mathbf{M}$	the determinant of $\mathbf{M}$	$\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$
$\operatorname{tr}\mathbf{M}$	the trace: sum of the diagonal entries	$\operatorname{tr}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = a + d$
λ	conventionally, an eigenvalue	$\mathbf{Mv} = \lambda\mathbf{v}$

Probability and statistics

Symbol	Meaning	Example or reading
$\mathrm{P}(A)$	the probability of the event A	
$\mathrm{P}(A \mid B)$	the probability of A given B	$\mathrm{P}(A \mid B) = \mathrm{P}(A \cap B)/\mathrm{P}(B)$; "$A$ given B"
$\mathrm{E}(X)$	the expectation (mean) of X	also written μ
$\mathrm{Var}(X)$	the variance of X	$\mathrm{E}(X^2) - \mathrm{E}(X)^2$; also written σ^2
$\mathrm{Cov}(X, Y)$	covariance	$\mathrm{E}(XY) - \mathrm{E}(X)\mathrm{E}(Y)$
$\mathrm{Corr}(X, Y)$	correlation	$\mathrm{Cov}(X, Y)/\sqrt{\mathrm{Var}(X)\mathrm{Var}(Y)}$; also written ρ
$X \sim \ldots$	X has the distribution ...	$X \sim \mathrm{B}(16, \frac{1}{2})$; "$X$ is distributed as ..."
$\mathrm{B}(n, p)$	binomial distribution	number of successes in n trials
$\mathrm{N}(\mu, \sigma^2)$	normal distribution, mean μ, variance σ^2	
$\mathrm{f}(x)$, $\mathrm{F}(x)$	probability density function; cumulative distribution function	$\mathrm{F}(x) = \mathrm{P}(X \le x)$
$\mathrm{G}(t)$	probability generating function	$\mathrm{G}(t) = \mathrm{E}(t^X)$
$\mathrm{M}_X(\theta)$	moment generating function	$\mathrm{M}_X(\theta) = \mathrm{E}(\mathrm{e}^{\theta X})$
$\bar{x}$	the mean of a sample	$\bar{x} = \frac{1}{n}\sum x_i$
$\hat{\theta}$	an estimate of the unknown quantity θ	"theta hat"

The Greek alphabet

Greek letters are used throughout mathematics, often with conventional meanings. The table gives the lower-case and capital forms (capitals that look like Roman letters, such as A for alpha, are omitted, since they are not used as separate symbols), the name, and some typical uses.

Letter	Name	Typical uses
α	alpha	angles; roots of an equation
β	beta	angles; roots of an equation; the beta function
γ, Γ	gamma	Euler's constant $\gamma \approx 0.5772$; the gamma function Γ; in relativity, $\gamma = 1/\sqrt{1 - v^2/c^2}$
δ, Δ	delta	a small quantity; Δ for a change, or for a triangle or discriminant
ε, ϵ	epsilon	a small positive quantity; in mechanics, a phase angle
ζ	zeta	the Riemann zeta function, $\zeta(s) = \sum_{n=1}^{\infty} n^{-s}$ for $s > 1$ (extended to other values, except $s = 1$, by more advanced methods)
η	eta	efficiency; a small quantity
θ, Θ	theta	angles; an unknown parameter in statistics
ι	iota	rarely used
κ	kappa	curvature; constants
λ, Λ	lambda	eigenvalues; a parameter, such as a Poisson rate; wavelength
μ	mu	a mean; the coefficient of friction
ν	nu	frequency
ξ, Ξ	xi	a variable or random quantity
π, Π	pi	$\pi \approx 3.14159$; Π for a product
ρ	rho	density; correlation; distance from an origin
σ, Σ	sigma	standard deviation; Σ for a sum
τ	tau	time; torque
υ, Υ	upsilon	rarely used
ϕ, φ, Φ	phi	angles; the golden ratio $\phi = \frac{1}{2}(1 + \sqrt{5})$; rapidity; Φ for the standard normal distribution function
χ	chi	the χ^2 (chi-squared) distribution
ψ, Ψ	psi	a wave function in quantum mechanics
ω, Ω	omega	angular velocity or frequency; a cube root of unity; Ω for a sample space

Mechanics

Symbol	Meaning	Example or reading
g	acceleration due to gravity	about $9.8\,\mathrm{m\,s^{-2}}$ at the Earth's surface
G	the gravitational constant	force Gm_1m_2/r^2 between two masses
e	coefficient of restitution (in italics, not to be confused with e)	along the line of impact, speed of separation $= e\times$ speed of approach
ω	angular velocity; angular frequency	$\ddot{x} = -\omega^2 x$ for simple harmonic motion
I	moment of inertia	rotational energy $\frac{1}{2}I\omega^2$ (Section 11.7)
c	the speed of light	about $3 \times 10^8\,\mathrm{m\,s^{-1}}$
$\hbar$	Planck's constant divided by 2π	"h bar" (Section 8.6)